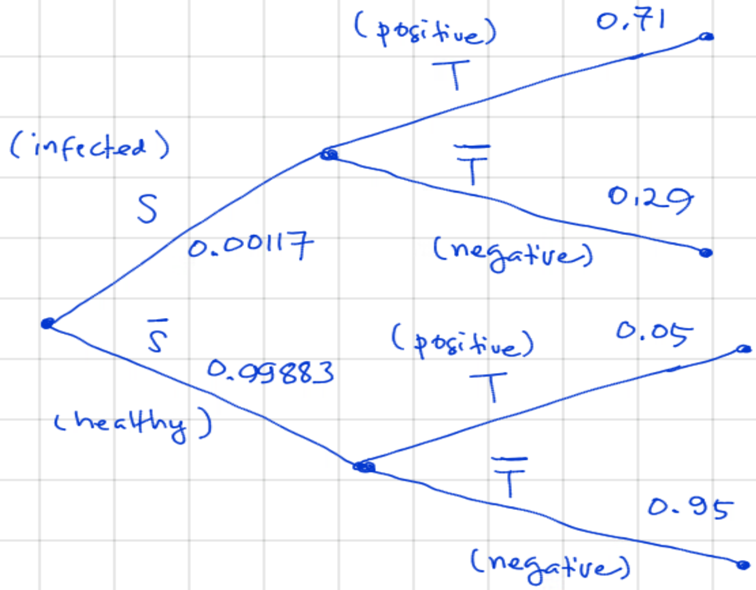
3. In the pandemic case of SARS-CoV-2 infections, testing for the virus is carried out using the swab test. Statistically said that there are 117 cases in 100,000 people have the disease. However, the test accuracy of the infected patient is not fully accurate. The test might return a 5% false positive, saying that from 100 non-infected people tested, 5 of them will be diagnosed to have the disease. On the other hand, if he/she is infected, the test might return 29% false negative, which means from 100 positively infected people, 29 of them will be diagnosed to be healthy. Using Bayes rule, calculate the probability that someone has the disease given the test is negative and the probability that someone has no disease despite having a positive test result. Show the steps in your answer.



$P(S) = 0.00117$

$P(\bar{S}) = 0.99883$

$P(T|S) = 0.71$

$P(\bar{T}|S) = 0.29$

$P(T|\bar{S}) = 0.05$

$P(\bar{T}|\bar{S}) = 0.95$

Total probability of having positive test:

$$P(T) = P(T|S) \cdot P(S) + P(T|\bar{S}) \cdot P(\bar{S})$$
$$= (0.71)(0.00117) + (0.05) \cdot (0.99883)$$
$$= 0.050772$$

Total probability of having negative test:

$$P(\bar{T}) = P(\bar{T}|S) \cdot P(S) + P(\bar{T}|\bar{S}) \cdot P(\bar{S})$$
$$= (0.29)(0.00117) + (0.95)(0.99883)$$
$$= 0.949228$$

The probability of someone is infected given that the test is negative:

$$P(S|\bar{T}) = \frac{P(\bar{T}|S) \cdot P(S)}{P(\bar{T})} = \frac{(0.29)(0.00117)}{0.949228} = 0.035\%$$

The probability of someone is healthy given that the test is positive:

$$P(\bar{S}|T) = \frac{P(T|\bar{S}) \cdot P(\bar{S})}{P(T)} = \frac{(0.05)(0.99883)}{0.050772} = 98.37\%$$

4. Table 2 shows a sample of an encrypted semiconductor dataset. The dataset is developed after the feature selection process. This dataset consists of two feature inputs, f_1 and f_2, that represent two normalized parameters and one categorical parameter that represent the condition of the semiconductor. Categorical refers to testing flag that is assigned in categorical form of "1" that refers to good units, whereas "0" refers to faulty units.

Table 2. Semiconductor dataset

f_1	f_2	*CLASS*
2	5	1
3	7	1
4	6	1
4	4	0
5	5	0
6	4	0

Determine how the Gaussian Naive Bayes classifier would classify the class of a new semiconductor product with $f_1 = 4$ and $f_2 = 5$.

f_1	f_2	Class	
2	5	1	(good)
3	7	1	(good)
4	6	1	(good)
4	4	0	(bad)
5	5	0	(bad)
6	4	0	(bad)

$P(\text{good}) = 3/6 = 0.5$

$P(\text{bad}) = 3/6 = 0.5$

As this is a continuous dataset, we will evaluated based on the probability density function /PDF (not the probability itself). Assuming normal distribution, PDF involves 2 parameters, ie. mean and standard deviation. Therefore, we need to calculate those for each input and class.

$\mu_{f1,\text{good}} = \frac{2+3+4}{3} = 3$; $\sigma_{f1,\text{good}} = \sqrt{\frac{(2-3)^2 + (3-3)^2 + (4-3)^2}{3-1}} = 1$

$\mu_{f2,\text{good}} = 6$; $\sigma_{f2,\text{good}} = 1$

$\mu_{f1,\text{bad}} = 5$; $\sigma_{f1,\text{bad}} = 1$

$\mu_{f2,\text{bad}} = 4.333$; $\sigma_{f2,\text{bad}} = 0.57735$

For the new individual $X(f_1 = 4, f_2 = 5)$, the class can be defined by calculating the probabilities of the two classes. (please note that we are actually not calculating the probability here but calculating the probability density)

$\Longrightarrow$

Probability of class 'good' given the new individual X :

$$P(good|X) = \frac{P(X|good) \cdot P(good)}{P(X|bad) \cdot P(bad) + P(X|good) \cdot P(good)}$$

while the probability of class 'bad' given the individual X:

$$P(bad|X) = \frac{P(X|bad) \cdot P(bad)}{P(X|bad) \cdot P(bad) + P(X|good) \cdot P(good)}$$

① To determine the class, we will compare the two above values. But as they have the same denominator, we just have to compare the numerators.

② Instead of calculating the probability, as we are dealing with continuous datasets, we will calculate the probability density.

$$\Rightarrow P(X|good) \cdot P(good) = P(f_{1,x}|good) \cdot P(f_{2,x}|good) \cdot P(good)$$
$$\propto PD(f_{1,x}|good) \cdot PD(f_{2,x}|good) \cdot P(good)$$

{PD = probability density}

$$= \frac{1}{\sigma_{f_1,good}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\overset{f_{1,x}}{\overset{\downarrow}{4}}-\mu_{f_1,good}}{\sigma_{f_1,good}}\right)^2} \cdot \frac{1}{\sigma_{f_2,good}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\overset{f_{2,x}}{\overset{\downarrow}{5}}-\mu_{f_2,good}}{\sigma_{f_2,good}}\right)^2} \cdot P(good)$$

$$= (0.2420)\ (0.2420) \cdot (0.5) = 0.0293$$

$$\Rightarrow P(X|bad) \cdot P(bad) = P(f_{1,x}|bad) \cdot P(f_{2,x}|bad) \cdot P(bad)$$

$$\propto \frac{1}{\sigma_{f_1,bad}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{4-\mu_{f_1,bad}}{\sigma_{f_1,bad}}\right)^2} \cdot \frac{1}{\sigma_{f_2,bad}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{5-\mu_{f_2,bad}}{\sigma_{f_2,bad}}\right)^2} \cdot P(bad)$$

$$= (0.2420)\ (0.3548)\ (0.5) = 0.0429$$ (LARGER!)

$\Rightarrow$ X belongs to 'bad' class.

5. A dataset containing two inputs, x_1 and x_2, and one output, t, has been collected. An Artificial Neural Network (ANN) with a structure shown in Figure 1 is used to model the data. Please note as an activation function, a sigmoid function, $a(z) = \frac{1}{1+exp^{-\alpha t}}$, is used and there is no nonlinear activation function in the output layer.

   a. derive the backpropagation functions for all updating weights, w_i and b_j for $i = 1..5$ and $j = 1..2$, to fit the model to capture the target output, t.

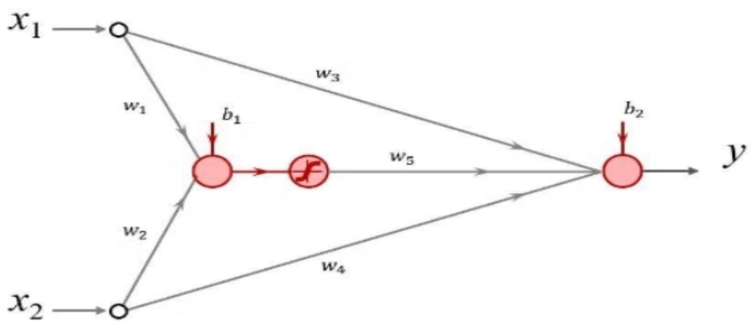
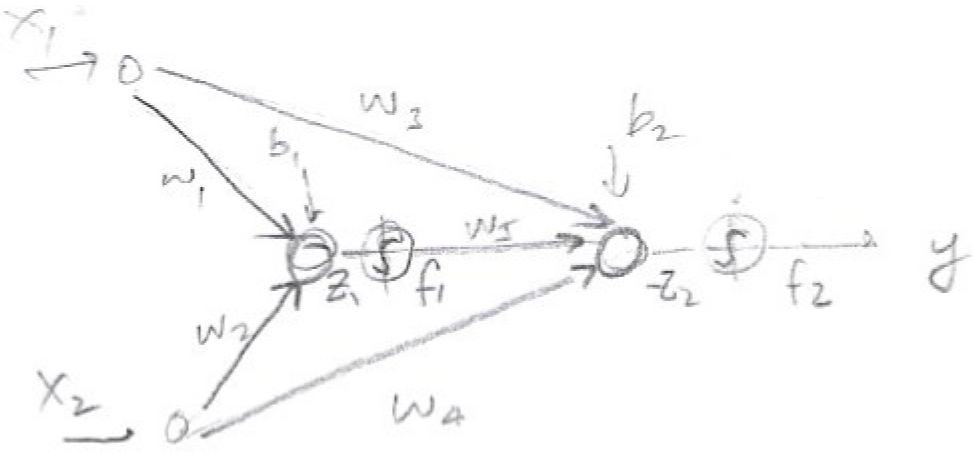


Figure 1. An artificial neural network for Q5



$E = \frac{1}{2}(t-y)^2 \; ; \; y = f(z_2)$

$y = f(w_3 x_1 + w_4 x_2 + w_5 f_1 + b_2)$

$y = f[w_3 x_1 + w_4 x_2 + w_5 f(w_1 x_1 + w_2 x_2 + b_1) + b_2]$

$$\frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial y} \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial b_2} = -(t-y) \cdot f'(z_2) \cdot$$

$$\frac{\partial E}{\partial w_5} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_5} = -(t-y) \cdot f'(z_2) \cdot f_1$$

$$\frac{\partial E}{\partial w_4} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_4} = -(t-y) \cdot f'(z_2) \cdot x_2$$

$$\frac{\partial E}{\partial w_3} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_3} = -(t-y) \cdot f'(z_2) \cdot x_1$$

$$\frac{\partial E}{\partial b_1} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial f_1} \cdot \frac{\partial f_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial b_1}$$

$$= -(t-y) \cdot f'(z_2) \cdot w_5 \cdot f'(z_1)$$

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial f_1} \cdot \frac{\partial f_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_2}$$

$$= -(t-y) \cdot f'(z_2) \cdot w_5 \cdot f'(z_1) \cdot x_2$$

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial z_2} \cdot \frac{\partial z_2}{\partial f_1} \cdot \frac{\partial f_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_1}$$

$$= -(t-y) \cdot f'(z_2) \cdot w_5 \cdot f'(z_1) \cdot w_1$$